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Comparative Analysis and Membership Optimization for Clustering Process in Machine Intelligence

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Abstract **-Fuzzy clustering is superior to crisp clustering when the boundaries among the clusters are vague and ambiguous. However, the main limitation of both fuzzy and crisp clustering algorithms is their sensitivity to the number of potential clusters and/or their initial positions. Moreover, the comprehensibility of obtained clusters is not expertized, whereupon in data-mining applications, the discovered knowledge is not understandable for human users. To overcome these restrictions, a novel fuzzy rule-based clustering algorithm (FRBC) is proposed. Like fuzzy rule-based classifiers, the FRBC employs a supervised classification approach to do the unsupervised cluster analysis. It tries to automatically explore the potential clusters in the data patterns and identify them with some interpretable fuzzy rules. Simultaneous classification of data patterns with these fuzzy rules can reveal the actual boundaries of the clusters. Compared with hard clustering, fuzzy clustering that allows overlap between clusters is able to provide a more accurate and natural description of the underlying structure of real-world data. The same as k-means, most existing studies on fuzzy clustering, including the well-known fuzzy c-means (FCM) and some recently proposed approaches are deals with vector-based data, of which each object is represented as a vector in some feature space. Research has shown fuzzy c-means (FCM) clustering to be a powerful tool to partition samples into different categories. However, the objective function of FCM is based only on the sum of distances of samples to their cluster centers, which is equal to the trace of the within-cluster scatter matrix. Hence a clustering algorithm based on both within and between-cluster scatter matrices, extended from linear discriminant analysis (LDA), and its application to an unsupervised feature extraction (FE). LDA methods comprise between and within- cluster scatter matrices modified from the between- and within-class scatter matrices of LDA.**

Index term- **FCM Clustering, Fuzzy Rule Based Clustering, Linear Discriminant Analysis (LDA).**

I. INTRODUCTION

Clustering has been a fundamental and efficient tool for data analysis by grouping similar objects into clusters. Compared with hard clustering, fuzzy clustering that allows overlap between clusters is able to provide a more accurate and natural description of the underlying structure of real-world data. The same as *k*-means, most existing studies on fuzzy clustering, including the well-known fuzzy c-means (FCM) and some recently proposed approaches, deal with vector-based data, of which each object is represented as a vector in some feature space. Clustering analysis has the potential to detect underlying structures within data, for classification and pattern recognition, as well as for model reduction and optimization. Fuzzy clustering, i.e., the synthesis between clustering and fuzzy set theory, is suitable to handle problems with vague boundaries of clusters. In fuzzy clustering, the requirement that each object is assigned to only one cluster is relaxed to a weaker requirement in which the object can belong to all of the clusters with a certain degree of membership. Moreover, the memberships may help us to discover more sophisticated relations between a given object and the disclosed clusters. It can also provide a basis for the construction of a rule-based fuzzy model that is human readable and performs well for nonlinear problems.

Different from conventional clustering approaches that generate clusters of objects of the same type based on the vector representation or the pair-wise relation, coclustering approaches simultaneously cluster the rows and columns of a data matrix. For example, both document clusters and word clusters are produced in document coclustering based on the document–word co-occurrence matrix. Co-clustering approaches have been proposed initially to handle high-dimensional data, such as text documents and micro array data, where the effectiveness of traditional distance-based clustering approaches degrades because of the "curse of dimensionality".

Fuzzy c-means (FCM) is one of the most well-known clustering methods. However, the objective function of FCM is only based on the sum of distances between samples to their cluster centers, which is equal to the trace of the within-cluster scatter matrix. In recent years, linear discriminant analysis (LDA) has often been used for dimensional reduction in supervised classification problems.

II. BACKGROUND

In this paper, A Fuzzy Rule-Based Clustering Algorithm (FRBC) [1], A Fuzzy Clustering Approach for Multi-type Relational Data Clustering (FC-MR) [2] and Linear Discriminant Analysis (LDA) Based Algorithm, and its application to an unsupervised feature extraction (LDA) [3] are reviewed.

A fuzzy rule-based classification system [1] is a special case of fuzzy modeling, where the output of system is crisp and discrete. The fuzzy models present two main advantages. First, they permit working with imprecise data, and they provide a comfortable way to naturally represent the missing values. Second, the acquired knowledge with these models may be more human understandable. Impressing the second preference of (FRBC), this tries to automatically explore the potential clusters in the data patterns with no prior knowledge. The obtained clusters are specified with some interpretable fuzzy rules, which make the clusters understandable for humans. The data in real-world data-mining applications may contain relations that involve more than two types. For example, for a dataset about published research papers, other than knowing a set of words each paper contains, we may also know the authors of each paper, the name of conferences or journals where those papers are published, and the references of each paper. This dataset can be considered as four-type relational data. It consists of *paper*, *term*, *author*, and *venue* (e.g., conference, journal) these four types of objects or entities, which form four relations i.e., *paper–term* relation, *paper–author* relation, *paper–venue* relation, and *paper–paper* relation. The first three relations characterize each individual paper with respect to different aspects, namely, the content, the person(s) who writes the paper, and the place where the paper is published, respectively, while the last relation records the citation information of one paper to another paper. A small research paper dataset is given in Fig. 1 to illustrate the relations among multiple object types.

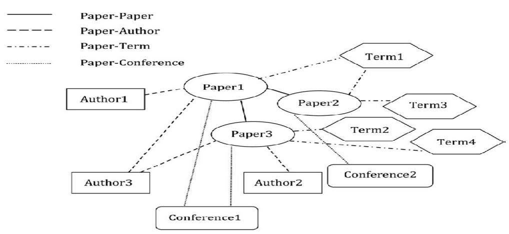

Fig.1 Four different relations in a small research paper dataset. The dataset involves three papers, three authors, four terms, and two conferences.

Multitype relational data may form various structures, depending on the availability of relations. A star-structure is a special case where relations only exist between the central type and several attribute types. For example, the research paper dataset forms a four-type star-structure with the *paper–term*, *paper– author*, and *paper–venue* relations, where *paper* is the central type and *term*, *author*, and *venue* are three attribute types. It is not a star-structure any more when the *paper–paper* relation is considered. It is possible to transform a multitype relational data into one of the basic data representation forms and then use an existing approach to get the clusters of objects of the interested type. However, useful information may be lost during data transformation. Moreover, clustering on each type of objects individually loses the chance of mutual improvement among clusters of different object types and is unable to capture the interrelated patterns among different types which may be of interest in some data-mining applications. To effectively make use of those relations among multiple object types researchers began to explore multi-type relational data clustering approaches [2], which produce clusters for different types simultaneously.

Despite the considerable efforts made by the clustering community, the common characteristics of distance functions suitable for FCM remain unclear. To fill this crucial void, a generalization of distance function for FCM with centroid of arithmetic mean [8] proposed, which provide a generalized definition of distance functions that fit FCM directly. In recent a LDA-Based clustering algorithm based on the Fisher criterion [3] is proposed which is reviewed in this paper. *K*-means is a partitional clustering technique that is well-known and widely used for its low computational cost. The K-means type algorithms versus imbalance data distributions [4] include the hard *k*-means and the fuzzy *k*-means. However, the performance of these algorithms tends to be affected by skewed data distributions, i.e., imbalanced data. They often produce clusters of relatively uniform sizes, even if input data have varied cluster sizes, which are called the “uniform effect.” To prevent the effect of the “uniform effect,” This method [4] propose a multicenter clustering algorithm in which multi-centers are used to represent each cluster, instead of one single center. Clustering algorithm and cluster validity are two highly correlated parts in cluster analysis. A novel idea for cluster validity and a clustering algorithm based on the validity index [5] are proposed. However, the cost of storing and manipulating the complete kernel matrix makes it infeasible for large problems. The Nystrom method [6] is a popular sampling-based low-rank approximation scheme for reducing the computational burdens in handling large kernel matrices. There are some variants of the widely used Fuzzy C-Means (FCM) algorithm that support clustering data distributed across different sites. Those methods have been studied under different names, like collaborative and parallel fuzzy clustering. Some augmentation of the two FCM-based clustering algorithms are done in [7], used to cluster distributed data by arriving at some constructive ways of determining essential parameters of the algorithms (including the number of clusters) and forming a set of systematically structured guidelines such as a selection of the specific algorithm depending on the nature of the data environment and the assumptions being made about the number of clusters.

Pattern recognition in real-world data is subject to various sources of uncertainty that should be appropriately managed. Recently general type-2 fuzzy c-means algorithm for uncertain fuzzy clustering algorithm [9] proposed. The focus of this algorithm is the management of uncertainty associated with parameters of fuzzy clustering algorithms. Type-2 fuzzy sets (T2 FSs) have received increased research interest over the past decade, primarily due to their potential to model various uncertainties. Clustering on uncertain data, one of the essential tasks in mining uncertain data, posts significant challenges on both modeling similarity between uncertain objects and developing efficient computational methods.

The rest of this paper is organized as follows. In Section 3, we discuss previous works done on various methodologies, and in Section 4, we discuss the analysis and discussion. We present the proposed methodology in Section 5, and we present possible outcome and results in Section 6. We conclude this paper in Section 7.

III. PREVIOUS WORK DONE

Over the past decades, many clustering algorithms have been proposed. The fuzzy *c*-means (FCM) is the best-known and the most popular fuzzy clustering algorithm whose time requirement is high. It also suffers from the presence of noise and outliers and the difficulty to identify the initial partitions. Moreover, it supposes that the points in the dataset are equally important and all clusters are spherical. Additionally, it assumes that there are almost equal numbers of points in the clusters. Therefore, many extensions to the FCM algorithm had been proposed in the literature. Some approaches have been proposed to improve its performance, as well as to decrease its computational complexity. Wang *et al.* proposed a center initialization approach that is based on a minimum spanning tree to keep the FCM from local minima. A possibilistic FCM (PFCM) model is presented in as a hybridization of possibilistic *c*-means (PCM) and FCM, which produces memberships and possibilities, simultaneously along with the cluster centers, and tries to solve the noise sensitivity defect of the FCM. A fuzzy rule-based classification system [1] is a special case of fuzzy modeling, where the output of system is crisp and discrete. The fuzzy models present two main advantages. First, they permit working with imprecise data, and they provide a comfortable way to naturally represent the missing values. Second, the acquired knowledge with these models may be more human understandable. Impressing the second preference in a fuzzy rule-based clustering algorithm (FRBC) [1], this tries to automatically explore the potential clusters in the data patterns with no prior knowledge. The obtained clusters are specified with some interpretable fuzzy rules, which make the clusters understandable for humans. Co-clustering approaches have been proposed initially to handle high-dimensional data, such as text documents and micro array data, where the effectiveness of traditional distance-based clustering approaches degrades because of the “curse of dimensionality.” Co-clustering is actually bitype heterogeneous relational data clustering or two-way clustering as it produces clusters for two object types simultaneously. But in a new fuzzy clustering approach for multi-type relational data (FC-MR) [3] different types of objects are clustered simultaneously. An object is assigned a large membership with respect to a cluster if its related objects in this cluster have high rankings.

Thus in this paper, A fuzzy rule-based clustering [1], A Fuzzy Clustering Approach for Multi-type Relational Data Clustering (FC-MR) [2] and Linear Discriminant Analysis (LDA) Based Algorithm, and its application to an unsupervised feature extraction (LDA) [3] are briefly discussed for automatically explore the potential clusters in the data patterns with no prior knowledge and tries to optimized membership for clustering.

IV. Analysis And Discussion

4.1 Fuzzy Rule-Based Clustering Algorithm

To appropriately estimate the number of auxiliary data patterns should be added, the summation of a within-cluster point to point scatter matrix is used. This value for main data is defined as

$$q = \sum_{i<j} d(X_i, X_j)$$

Where *Xi* is one of the *mM* main data patterns, and *d*(· , ·) is a distance metric, which is usually the Euclidean norm. Similarly, for auxiliary data $\{X_i', i=1,2,\ldots m_A\}$

$$q' = \sum_{i<j} d(X_i', X_j').$$

By the use of *q* and *q'*, the auxiliary random patterns are added incrementally until *q'* exceeds *q*. In this regard, the distribution of main data patterns influences the size of auxiliary instances. On one hand, if the main data patterns are uniformly distributed in the problem space (i.e., not meaningful clusters can be explored), the same number of random instances might be added and these auxiliary data behave as the main data patterns in this case [see Fig. 2(a)]. On the other hand, when there are some clear clusters in the pattern space of the problem, the number of auxiliary instances would be less than the main data [e.g., Fig. 2(b)]. Some factors that influence this parameter are as follows: the scatterity of clusters (i.e., between-clusters distances), their condensity (i.e., within-cluster distances), the cardinality of clusters (i.e., size of clusters), and their sparseness (i.e., the distribution of clusters in the problem space).

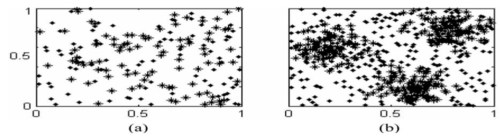

Fig.2 Two examples of main and auxiliary data patterns. (a) No cluster in main data. (b) Three clusters in main data.

After the preparation of the two-class problem that consists of main and auxiliary data patterns, the FRBC employs SGERD as a fuzzy classification rule generator to produce some fuzzy rules to solve this two-class problem.

After generation of the fuzzy rules, the FRBC sets aside the main data in the fuzzy subspace of the best rule as the initial members of the first cluster and removes them from the problem space to not be reconsidered for other clusters.

4.2 A Fuzzy Approach for Multi-type Relational Data Clustering

4.2.1 Problem Formulation

A dataset with *m* object types $x = \{\}^{m}_{\mu=1}$, where $X_\mu = \{x_i^\mu\}^n_{i=1}$ is a set of objects of the *μ*th type, and $n\mu = |X\mu|$.

To formulate this problem, for each type $\mu = \{1, 2 \ldots m\}$, here define a cluster membership u_{if}^{μ} and a ranking v_{if}^{μ} *if* for each object *xμi* with respect to each cluster *f*. The membership is a soft or fuzzy cluster indicator that measures how possible an object is labeled as that cluster, and the ranking measures how representative or typical *xμi* is compared with other objects of the same type in cluster *f*.

4.2.2 Solution

The method of Lagrange multiplier to derive the local solutions for the constrained optimization problem is used in proposed algorithm. The Lagrangian is formed as

$$L = J + \sum_{\mu=1}^{m} \vec{\gamma}_\mu^T (\mathbf{U}_\mu \mathbf{1} - \mathbf{1}) + \sum_{\nu=1}^{m} \vec{\lambda}_\nu^T (\mathbf{V}_\nu^T \mathbf{1} - \mathbf{1})$$
$$+ \sum_{\mu=1}^{m} \mathbf{1}^T (\boldsymbol{\alpha}_\mu \odot \mathbf{U}_\mu)\mathbf{1} + \sum_{\nu=1}^{m} \mathbf{1}^T (\boldsymbol{\beta}_\nu \odot \mathbf{V}_\nu)\mathbf{1}.$$

4.3 LDA-Based Clustering Algorithm and Its Application to an Unsupervised Feature Extraction.

This method is a novel clustering algorithm, i.e. LDA-based clustering (FLDC), and its application to an unsupervised FE and UFLDA [3].

Two unsupervised between- and within-cluster scatter matrices are derived from the scatter matrices of LDA and are applied to formulate FLDC. Here define the between-cluster scatter matrix S_b^{UFLDA} and the within-cluster scatter matrix S_w^{UFLDA} as follows:

$$S_b^{\mathrm{UFLDA}} = \sum_{i=1}^{L} \frac{\sum_{j=1}^{N} u_{ij}}{N} (c_i - c)(c_i - c)^T$$

$$S_w^{\mathrm{UFLDA}} = \sum_{i=1}^{c} \sum_{j=1}^{N} \frac{u_{ij}}{N} (x_j - c_i)(x_j - c_i)^T$$

Where $C_i = \sum_{j=1}^{N} (u_{ij} / \sum_{i=1}^{N} u_{ik}) x_j$ is the class mean, which is the same as FCM, and $c = 1/N \sum_{i=1}^{N} x_j$ represents the total mean.

A brief comparison of a Fuzzy Rule-Based Clustering Algorithm (FRBC) [1], a Fuzzy Clustering Approach for Multi-type Relational Data Clustering (FC-MR) [2] and Linear Discriminant Analysis (LDA) Based Algorithm, and its application to an unsupervised feature extraction (LDA) [3] algorithms are as shown in Table 1.

Clustering methods	**Advantages**	**Disadvantages**
Fuzzy Rule-Based Clustering Algorithm (FRBC)	1. The generated fuzzy rules, which represent the clusters, are human understandable. 2. It tries to automatically explore the potential clusters in the data patterns and identify them with some interpretable fuzzy rules.	1. Although the clustering ability of the FRBC applying on synthetic and real-world datasets, but less clusters than classes may be explored for multiclass datasets. 2. The application of the FRBC on large datasets consumes more CPU time.
Fuzzy Clustering Approach for Multi-type Relational Data Clustering (FC-MR)	1. The FC-MR is able to label all the documents correctly on all the datasets. 2. The proposed FC-MR is able to handle relational data with various structures.	1. FC-MR for standard star-structure only consists of relations between the central type and the attribute types.
Linear Discriminant Analysis (LDA) Based Algorithm	1. The results of clustering FLDC significantly outperformed. 2. The FLDC had the best performance with the regular and noisy lines datasets.	1. FLDC only worked well when the distribution of clusters showed normal-like distribution.

Table 1: Comparison between FRBC, FCMR and LDA-Based Clustering Algorithm

V. PROPOSED METHODOLOGY

Fuzzy clustering, i.e., the synthesis between clustering and fuzzy set theory, is suitable to handle problems with vague boundaries of clusters . In fuzzy clustering, the requirement that each object is assigned to only one cluster is relaxed to a weaker requirement in which the object can belong to all of the clusters with a certain degree of membership. Moreover, the memberships may help us to discover more sophisticated relations between a given object and the disclosed clusters. It can also provide a basis for the construction of a rule-based fuzzy model that is human readable and performs well for nonlinear problems.

For this purpose, first each attribute is rescaled to unit interval [0, 1] by the usage of a linear transformation that preserves the distribution of training patterns. Then, the pattern space is partitioned into fuzzy subspaces, and each subspace is identified by a fuzzy rule, if there are some patterns in that subspace. To do partitioning, usually, K suitable membership functions are used to assign K linguistic values to each input attribute. Traditionally, triangular membership functions are used for this purpose, because they are simpler and more human understandable.

Fig. 3 shows these membership functions for four different values of K, where the linguistic labels $L3$, $L4$, and $L5$, for example, can be interpreted as linguistic values *small*, *medium*, and *large*, respectively.

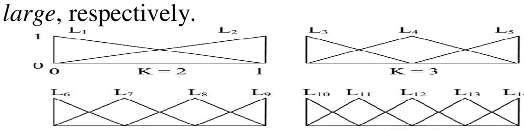

Fig3. Different partitioning of each input attribute.

Given an input partitioning of the pattern space, there are several approaches to generate fuzzy classification rules from data.

VI. POSSIBLE OUTCOMES AND RESULT

In this section, we have compared various fuzzy clustering algorithms in terms of accuracy and performance according to their possible outcomes and results.

The clustering results of the application of the FRBC on 11 classification datasets are shown in Table 2, where comparison of the computational cost of the FRBC with that of the FCM is given.

Data set	CPU time (seconds)	
	FRBC	FCM
Iris	1.0	0.17
Thyroid	0.2	0.10
Ecoli	1.2	0.10
Cancer	7.6	0.10
Glass	1.0	0.10
Vowel	24.0	0.17
Wine	1.0	0.10
Vehicle	6.0	0.14
WDBC	2.2	0.15
Ionosphere	3.2	0.10
Sonar	1.8	0.20

Table 2. Computational Cost of FRBC and FCM

The original data of 20newsgroups1 contains 18 828 non-duplicated documents, which are categorized into 20 topics. Here generate three subsets that consist of different subtopics, as listed in Table3 below.

TM1	C_1: {rec.sport.baseball, rec.sport.hockey}
	C_2: {talk.politics.guns, talk.politics.mideast, talk.politics.misc}
TM2	C_1: {comp.graphics, comp.os.ms-windows.misc}
	C_2: {rec.autos, rec.motorcycles}
	C_3: {sci.crypt, sci.electronics}
TM3	C_1: {comp.sys.ibm.pc.hardware, comp.sys.mac.hardware}
	C_3: {rec.autos, rec.motorcycles}
	C_3: {sci.med, sci.space}
	C_4: {talk.politics.guns, talk.politics.mideast}

Table3. Structure of Newsgroup Dataset

FC-MR gives the highest Accuracy and NMI values among the five approaches on all the three datasets, which is shown as Table4 below.

Accuracy			
	TM1	TM2	TM3
HFCM	97.32 ± 10.21	89.49 ± 14.12	86.30 ± 14.76
FCoDok	97.32 ± 10.21	89.49 ± 14.12	86.71 ± 14.23
NMF	97.26 ± 10.20	86.30 ± 9.38	87.53 ± 13.88
SRC	**100.00 ± 0.00**	75.88 ± 13.96	77.57 ± 22.86
FC-MR	**100.00 ± 0.00**	**99.16 ± 4.61**	**95.52 ± 11.78**
NMI			
	TM1	TM2	TM3
HFCM	97.32 ± 24.89	89.49 ± 18.57	86.30 ± 14.34
FCoDok	97.32 ± 24.89	89.49 ± 18.57	86.71 ± 14.54
NMF	97.26 ± 24.80	86.30 ± 13.83	87.53 ± 12.40
SRC	**100.00 ± 0.00**	72.06 ± 3.82	85.85 ± 15.87
FC-MR	**100.00 ± 0.00**	**99.16 ± 7.23**	**95.52 ± 12.03**

Table4. Comparison of Accuracy and NMI on Newsgroup Data

These results show that FC-MR achieves significant improvement in document clustering compared with existing vector-based fuzzy clustering and fuzzy co-clustering with combined relations; it also performs much better than nonnegative matrix-factorization based and spectral-clustering-based multi-type relational data clustering approaches.

CONCLUSION

This paper exhausted different clustering algorithm for optimizing procedure and estimate fuzzy memberships of objects in each cluster. The FRBC, i.e. a novel fuzzy rule-based clustering algorithm automatically explore the potential clusters in the datasets. It looks at the clustering issue as a classification problem by the addition of some auxiliary data patterns to the main data and then generation of some fuzzy rules to classify the new pattern space. The main idea of the FC-MR approach is to estimate fuzzy memberships of objects in each cluster based on the ranking values of related objects in that cluster, and the memberships are, in turn, used to update the rankings of objects in each cluster. The FC-MR is able to handle relational data with various structures. FC-MR can be applied in various situations according to the requirement of the applications and the availability of relations in the data. The FLDC method composed of the between- and within cluster scatter matrices extended from LDA and its application for an unsupervised FE, UFLDA.

FUTURE SCOPE

Although the clustering ability of the FRBC applying on synthetic and real-world datasets, but less clusters than classes may be explored for multiclass datasets. This can be obviated by decreasing the value of threshold τ for the rule effectiveness measure. Intuitively, for an unknown dataset, the FRBC should compute this measure for all potential clusters, and when a sudden drop occurs in their values, it should be pretended as the stopping condition, which determines the number of clusters that can be explored. The future work on the FRBC should turn to this issue more seriously. Upcoming direction in research shows that the LDA optimization problem is non-convex and nonlinear. Next study will be to develop or choose an appropriate criterion for FLDC (Akaike and Bayesian information criteria) to determine the number of clusters.

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